14.4.1 For the functions $w = 9x^2 + 3y^2$, $x = \cos t$, and $y = \sin t$. express $\frac{dw}{dt}$ as a function of t, both by using the chain rule and by expressing w in terms of t and differentiating directly with respect to t. Then, evaluate $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.

Express $\frac{dw}{dt}$ as a function of t.

Partial differentiate w with respect to x

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}(9x^2 + 3y^2) \quad \text{treat } y \text{ as a constant}$$

$$\frac{\partial w}{\partial x} = 18x + 0$$

$$= 18x$$

Partial differentiate w with respect to y

$$\frac{\partial w}{\partial y} = \frac{\partial}{\partial y}(9x^2 + 3y^2) \quad \text{treat } x \text{ as a constant}$$

$$\frac{\partial w}{\partial y} = 0 + 6y$$

$$= 6y$$

differentiate $x = \cos t$ with respect to t

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$= -\sin t$$

differentiate $y = -\sin t$ with respect to t

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$= \cos t$$

write $\frac{dw}{dt}$ using the chain rule $\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}}$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} = 18x(-\sin t) + 6y(\cos t)$$

$$= -18x \sin t + 6y \cos t$$

$$= -18(\cos t)\sin t + 6(\sin t)\cos t \quad \text{plug } x = \cos t,\ y = \sin t$$

$$\frac{dw}{dt} = -12 \sin t \cos t$$

Evaluate $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.

$$\left.\frac{dw}{dt}\right|_{t=\frac{\pi}{2}} = -12 \sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)$$

$$= -12(1)\cdot 0 \quad \text{since } \sin\frac{\pi}{2} = 1,\ \cos\frac{\pi}{2} = 0.$$

$$= 0$$

14.4.7 consider the functions $z = -3e^x \ln y$, $x = \ln(u\cos v)$, and $y = u\sin v$.

a) express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of both u and v both by using the chain rule and by expressing z directly in terms of u and v before differentiating.

b) evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(u,v) = \left(3, \frac{\pi}{3}\right)$

a) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$

Partial differentiate z with respect to x

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(-3e^x \ln y) = -3e^x \ln y$$

Partial differentiate x with respect to u

$$\frac{\partial x}{\partial u} = \frac{\partial}{\partial u}(\ln(u\cos v)) = \frac{1}{u\cos v}\cdot \cos v = \frac{\cancel{\cos} v}{u\cancel{\cos} v} = \frac{1}{u}$$

Chain rule $\frac{\partial}{\partial x}(\ln x) = \frac{1}{x}$

afuera • adentro

Partial differentiate z with respect to y

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(-3e^x \ln y) = \frac{-3e^x}{y}$$

Partial differentiate y with respect to u

$$\frac{\partial y}{\partial u} = \frac{\partial}{\partial u}(u\sin v) = \sin v$$

$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$

already have value of $\frac{dz}{dx}$ ... $-3e^x \ln y$

Partial differentiate x with respect to v

$$\frac{\partial x}{\partial v} = \frac{\partial}{\partial v}(\ln(u\cos v)) = \frac{1}{u\cos v}\cdot(-u\sin v) = \frac{-\cancel{u}\sin v}{\cancel{u}\cos v} = -\tan v$$

Chain rule $\frac{\partial}{\partial x}(\ln x) = \frac{1}{x}$

already have value of $\frac{dz}{dy}$ ... $\frac{-3e^x}{y}$

Partial differentiate y with respect to v

$$\frac{\partial y}{\partial v} = \frac{\partial}{\partial v}(u\sin v) = u\cos v$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = -3e^x \ln y\left(\frac{1}{u}\right) + \left(\frac{-3e^x}{y}\right)\sin v$$

$$= \frac{-3e^x \ln y}{u} - \frac{3e^x \sin v}{y}$$

insert values of x and y into equation

$$= -\frac{3e^{\ln(u\cos v)}\ln(u\sin v)}{u} - \frac{3e^{\ln(u\cos v)}\sin v}{u\sin v}$$

$e^{\ln x} = \ln$

$$= -\frac{3\cancel{u}\cos v \ln(u\sin v)}{\cancel{u}} - \frac{3\cancel{u}\cos v\cdot\cancel{\sin v}}{\cancel{u}\cancel{\sin v}}$$

$$= -3\cos v\cdot\ln(u\sin v) - 3\cos v$$

Factor $-3\cos v$

$$= \boxed{-3\cos v\,[1 + \ln(u\sin v)]}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = -3e^x\ln y(-\tan v) + \left(\frac{-3e^x}{y}\right)u\cos v$$

$$= 3e^x\ln y\cdot\tan v - \frac{3e^x\cdot u\cos v}{y}$$

$$= 3e^{\ln(u\cos v)}\ln(u\sin v)\cdot\tan v - \frac{3e^{\ln(u\cos v)}\cdot u\cos v}{u\sin v}$$

$$= 3u\cos v\ln(u\sin v)\tan v - \frac{3u\cos v\cdot\cancel{u}\cos v}{\cancel{u}\sin v}$$

$$= 3u\cancel{\cos v}\ln(u\sin v)\frac{\sin v}{\cancel{\cos v}} - \frac{3u\cos^2 v}{\sin v}$$

$$= \boxed{3u\ln(u\sin v)\sin v - \frac{3u\cos^2 v}{\sin v}}$$

b) evaluate $\frac{\partial z}{\partial u}$ at $(u,v) = \left(3, \frac{\pi}{3}\right)$

$$\left.\frac{\partial z}{\partial u}\right|_{(3,\pi/3)} = -3\cos v\left[1 + \ln(u\sin v)\right]$$

$$= -3\cos \pi/3 \left[1 + \ln(3\sin \pi/3)\right]$$

$$= -3 \cdot \frac{1}{2}\left[1 + \ln\left(3 \cdot \frac{\sqrt{3}}{2}\right)\right]$$

$$= -\frac{3}{2}\left(1 + \ln\frac{3\sqrt{3}}{2}\right)$$

evaluate $\frac{\partial z}{\partial v}$ at $(u,v) = \left(3, \frac{\pi}{3}\right)$

$$\left.\frac{\partial z}{\partial v}\right|_{(3,\pi/3)} = 3u\ln(u\sin v)\sin v - \frac{3u\cos^2 v}{\sin v}$$

$$= 3(3)\ln(3\sin \pi/3)\sin \pi/3 - \frac{3(3)\cos^2 \pi/3}{\sin \pi/3}$$

$$= 9\ln\left(3 \cdot \frac{\sqrt{3}}{2}\right)\frac{\sqrt{3}}{2} - \frac{9\left(\frac{1}{2}\right)^2}{\frac{\sqrt{3}}{2}}$$

$$= \frac{9\sqrt{3}}{2}\ln\frac{3\sqrt{3}}{2} - \frac{9\left(\frac{1}{2}\right)^2 \cdot 2}{\sqrt{3}}$$

Fraction rule $\frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$ $\quad \frac{x^a}{x^b} = x^{a-b}$ $\quad x^{a+b} = x^a x^b$

$$= \frac{9\sqrt{3}}{2}\ln\frac{3\sqrt{3}}{2} - \frac{18\left(\frac{1}{2}\right)^2}{\sqrt{3}}$$

$$\frac{9}{4} \cdot \frac{2}{\sqrt{3}} = \frac{18}{4\sqrt{3}}$$

$$= \frac{9\sqrt{3}}{2}\ln\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$\frac{18}{4\sqrt{3}} \cdot 4\sqrt{3} = \frac{72\sqrt{3}}{16(3)} = \frac{\cancel{72}\sqrt{3}}{48} = \frac{6\sqrt{3}}{4}$$

better way!

$$\frac{3}{2}\sqrt{3}$$

$$\frac{9\sqrt{3}\left(\frac{3}{2}\ln(3) - \ln(2)\right) - 3\sqrt{3}}{2}$$

14.4.25

assuming $4x^3 - y^2 - 3xy = 0$ defines y as a differentiable function of x, use the theorem $\frac{dy}{dx} = -\frac{F_x}{F_y}$ to find $\frac{dy}{dx}$ at the point $(1,1)$.

differentiate $F(x,y) = 4x^3 - y^2 - 3xy$ with respect to x.

$$F_x = \frac{\partial}{\partial x}\left(4x^3 - y^2 - 3xy\right) \quad \text{treat } y \text{ as a constant}$$

$$= 12x^2 - 0 - 3x$$

$$= 12x^2 - 3x$$

differentiate $F(x,y) = 4x^3 - y^2 - 3xy$ with respect to y.

$$F_y = \frac{\partial}{\partial y}\left(4x^3 - y^2 - 3xy\right) \quad \text{treat } x \text{ as a constant}$$

$$= 0 - 2y - 3xy$$

$$= -2y - 3xy$$

write $\frac{dy}{dx}$ using the theorem $\frac{dy}{dx} = -\frac{F_x}{F_y}$

$$\frac{dy}{dx} = -\frac{12x^2 - 3x}{-2y - 3xy}$$

substitute $x = 1$ and $y = 1$ and evaluate at the given point.

$$\left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{12(1)^2 - 3(1)}{-2(1) - 3(1)(1)} = \frac{9}{5}$$

14.4.31 if the equation $F(x,y,z) = 0$ determines z as a differentiable function of x and y, then, at the points where $F_z \neq 0$, the following equations are true.

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

use these equations to find the values of $\frac{\partial z}{\partial x}$ and $\frac{dz}{dy}$ at the given point.

$2z^3 - 4xy + 3yz + 3y^3 - 114 = 0$, $(4,3,3)$.

$$F_x = \frac{\partial}{\partial x}\left(2z^3 - 4xy + 3yz + 3y^3 - 114\right)$$
$$= 0 - 4y + 0 + 0 - 0$$
$$= -4y$$

$$F_y = \frac{\partial}{\partial y}\left(2z^3 - 4xy + 3yz + 3y^3 - 114\right)$$
$$= 0 - 4x + 3z + 9y^2 - 0$$
$$= -4x + 3z + 9y^2$$

$$F_z = \frac{\partial}{\partial z}\left(2z^3 - 4xy + 3yz + 3y^3 - 114\right) \longrightarrow F_z = 6z^2 + 3y$$
$$= 6z^2 - 0 + 3y + 0 + 0 - 0$$
$$= 6z^2 + 3y$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$
$$= -\frac{-4y}{6z^2 + 3y}$$
$$= \frac{4y}{6z^2 + 3y}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$
$$= -\frac{-4x + 3z + 9y^2}{6z^2 + 3y}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(4,3,3)} = \frac{4y}{6z^2 + 3y}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(4,3,3)} = -\frac{-4x + 3z + 9y^2}{6z^2 + 3y}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(4,3,3)} = \frac{4(3)}{6\cdot 3^2 + 3(3)}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(4,3,3)} = \frac{4\cdot 4 + 3\cdot 3 + 9(3)^2}{6\cdot 3^2 + 3\cdot 3}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(4,3,3)} = \frac{4}{21}$$

$$\left.\frac{\partial z}{\partial y}\right|_{(4,3,3)} = -\frac{74}{63}$$

Find $\frac{dw}{dr}$ when $r = 2$ and $s = -2$ if $w = (x+y+z)^2$, $x = r - s$, $y = \cos(r+s)$, $z = \sin(r+r)$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial x} = (x+y+z)^2 = 2(x+y+z)$$

$$\frac{\partial x}{\partial r} = r - s = 1$$

$$\frac{\partial w}{\partial y} = (x+y+z)^2 = 2(x+y+z)$$

$$\frac{\partial y}{\partial r} = \cos(r+s) = -\sin(r+s)$$

$$\frac{\partial w}{\partial z} = (x+y+z)^2 = 2(x+y+z)$$

$$\frac{\partial z}{\partial r} = \sin(r+s) = \cos(r+s)$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

Find $\frac{\partial w}{\partial r}$ when r = 5 and s = − 5 if $w = (x + y + z)^2$, x = r − s, y = **cos** (r + s), z = **sin** (r + s).

Use the Chain Rule to find $\frac{\partial w}{\partial r}$. The chain rule for two independent variables and three intermediate variables is shown below.

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

Evaluate each derivative separately. Differentiate w with respect to x. Treat y and z as constants.

$$\frac{\partial w}{\partial x} = 2(x + y + z)$$

Differentiate x with respect to r. Treat s as a constant.

$$\frac{\partial x}{\partial r} = 1$$

Differentiate w with respect to y. Treat x and z as constants.

$$\frac{\partial w}{\partial y} = 2(x + y + z)$$

Differentiate y with respect to r. Treat s as a constant.

$$\frac{\partial y}{\partial r} = -\sin(r + s)$$

Differentiate w with respect to z. Treat x and y as constants.

$$\frac{\partial w}{\partial z} = 2(x + y + z)$$

Differentiate z with respect to r. Treat s as a constant.

$$\frac{\partial z}{\partial r} = \cos(r + s)$$

Use the results from the derivatives above to write the expression for $\frac{\partial w}{\partial r}$.

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$
$$= 2(x + y + z) + 2(x + y + z)(-\sin(r + s)) + 2(x + y + z)\cos(r + s)$$

Now calculate the values of the intermediate variables x, y and z. To do this, substitute the values r = 5 and s = − 5 into the expressions x = r − s, y = **cos** (r + s), and z = **sin** (r + s). Start by calculating x.

$$\begin{aligned} x &= r - s \\ &= 5 - (-5) \\ &= 10 \end{aligned}$$

Calculate y.

$$\begin{aligned} y &= \cos(r + s) \\ &= \cos[5 + (-5)] \\ &= 1 \end{aligned}$$

Calculate z.

$$\begin{aligned} z &= \sin(r + s) \\ &= \sin[5 + (-5)] \\ &= 0 \end{aligned}$$

Substitute these values and the values of r and s into the expression for $\frac{\partial w}{\partial r}$ to solve.

$$\frac{\partial w}{\partial r} = 2(x + y + z) + 2(x + y + z)(-\sin(r + s)) + 2(x + y + z)\cos(r + s)$$

$$\begin{aligned} \left.\frac{\partial w}{\partial r}\right|_{r=5,\,s=-5} &= 2(10 + 1 + 0) + (2(10 + 1 + 0))(-\sin[5 + (-5)]) + 2(10 + 1 + 0)\cos[5 + (-5)] \\ &= 2(11) + 2(11)(0) + 2(11)(1) \\ &= 22 + 0 + 22 \\ &= 44 \end{aligned}$$

$$\begin{aligned} \frac{\partial w}{\partial r} &= 2(x+y+z)\cdot 1 + 2(x+y+z)\cdot -\sin(r+s) + 2(x+y+z)\cdot\cos(r+s) \\ &= 2(x+y+z) + 2(x+y+z)(-\sin(r+s)) + 2(x+y+z)\cos(r+s) \end{aligned}$$

$$\begin{aligned} x &= r - s \\ &= 2 - (-2) \\ &= 4 \end{aligned} \qquad \begin{aligned} y &= \cos(r+s) \\ &= \cos(2+(-2)) \\ &= \cos(0) \\ &= 1 \end{aligned} \qquad \begin{aligned} z &= \sin(r+s) \\ &= \sin(2+(-2)) \\ &= \sin(0) \\ &= 0 \end{aligned}$$

$$\frac{\partial w}{\partial r} = 2(x+y+z) + 2(x+y+z)(-\sin(r+s)) + 2(x+y+z)\cos(r+s)$$

$$\begin{aligned} \left.\frac{\partial w}{\partial r}\right|_{\substack{r=2\\ s=-2}} &= 2(4+1+0) + 2(4+1+0)(-\sin(2+(-2))) + 2(4+1+0)\cos(2+(-2)) \\ &= 2(5) + 2(5)(-\sin(0)) + 2(5)\cos(0) \\ &= 10 + 10(0) + 10(1) \\ &= 20 \end{aligned}$$

14.4.41 assume that $w = f(s^3 + t^2)$ and $f'(x) = e^x$. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

differentiate w with respect to t

$$\begin{aligned} \frac{\partial w}{\partial t} &= f(s^3 + t^2) \\ &= f'(s^3 + t^2)\frac{d}{dt}(s^3 + t^2) \\ &= 2t f'(s^3 + t^2) \end{aligned}$$

substitute $x = (s^3 + t^2)$ in $f'(x) = e^x$

$$f'(s^3 + t^2) = e^{s^3 + t^2}$$

substitute $f'(s^3 + t^2) = e^{s^3 + t^2}$ in $2t f'(s^3 + t^2)$

$$\begin{aligned} \frac{\partial w}{\partial t} &= 2t f'(s^3 + t^2) \\ &= 2t e^{s^3 + t^2} \end{aligned}$$

Differentiate w with respect to s,

$$\frac{\partial w}{\partial s} = f'(s^3 + t^2)\frac{d}{ds}(s^3 + t^2)$$

This implies that,

$$\frac{\partial w}{\partial s} = 3s^2 f'(s^3 + t^2) \quad \ldots\ldots (2)$$

Now substitute $f'(s^3 + t^2) = e^{s^3 + t^2}$ in equation (2), to get

$$\frac{\partial w}{\partial s} = 3s^2 e^{s^3 + t^2}$$

Therefore, $\boxed{\frac{\partial w}{\partial s} = 3s^2 e^{s^3 + t^2}}$

14.4.45 assume that $z = f(w)$, $w = g(x,y)$, $x = 3r^2 - s^2$, and $y = re^s$. if $g_x(3,1) = -4$, $g_y(3,1) = 2$, $f'(7) = -4$, and $g(3,1) = 7$, find the following

$$\left.\frac{\partial z}{\partial r}\right|_{r=1,\, s=0} \quad \text{and} \quad \left.\frac{\partial z}{\partial s}\right|_{r=1,\, s=0}$$

Assume that $z = f(w)$, $w = g(x,y)$, $x = 7r^3 - s^2$, and $y = re^s$. If $g_x(7,1) = -4$, $g_y(7,1) = 5$, $f'(5) = -5$, and $g(7,1) = 5$, find the following.

$$\left.\frac{\partial z}{\partial r}\right|_{r=1,\, s=0} \quad \text{and} \quad \left.\frac{\partial z}{\partial s}\right|_{r=1,\, s=0}$$

For functions z, f, and w, the derivative of $z = f(w)$ is $z' = f'(w) \cdot w'$.

Suppose that $w = f(x,y)$, $x = g(r,s)$, and $y = h(r,s)$. If all three functions are differentiable, then w has partial derivatives with respect to r and s, as given by the following formulas.

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} \quad \text{and} \quad \frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

Begin by finding $\frac{\partial w}{\partial r}$. Find each partial derivative with the respective variable. Notice that $\frac{\partial w}{\partial x} = g_x(7,1) = -4$. So begin by finding $\frac{\partial x}{\partial r}$.

$$x = 7r^3 - s^2$$
$$\frac{\partial x}{\partial r} = 21r^2$$

Next, find $\frac{\partial w}{\partial y}$ and $\frac{\partial y}{\partial r}$. Notice that $\frac{\partial w}{\partial y} = g_y(7,1) = 5$. So, find $\frac{\partial y}{\partial r}$.

$$y = re^s$$
$$\frac{\partial y}{\partial r} = e^s$$

Substitute these expressions into the formula and simplify.

$$\frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} = (-4)\left(21r^2\right) + (5)\left(e^s\right)$$
$$\frac{\partial z}{\partial r} = -84r^2 + 5e^s$$

Now evaluate $\frac{\partial z}{\partial r} = f'(w) \cdot \frac{\partial w}{\partial r}$ for $r = 1$ and $s = 0$.

$$\begin{aligned}\left.\frac{\partial z}{\partial r}\right|_{r=1,\, s=0} &= -5\left(-84r^2 + 5e^s\right) \\ &= -5\left[-84(1)^2 + 5e^0\right] \\ &= 395\end{aligned}$$

Use a similar process to find $\frac{\partial z}{\partial s}$. Recall that $\frac{\partial w}{\partial x} = g_x(7,1) = -4$ and $\frac{\partial w}{\partial y} = g_y(7,1) = 5$. Begin by finding $\frac{\partial x}{\partial s}$.

$$x = 7r^3 - s^2$$
$$\frac{\partial x}{\partial s} = -2s$$

Now find $\frac{\partial y}{\partial s}$.

$$y = re^s$$
$$\frac{\partial y}{\partial s} = re^s$$

Substitute these expressions into the formula and simplify.

$$\frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} = (-4)(-2s) + (5)\left(re^s\right)$$
$$\frac{\partial z}{\partial s} = 8s + 5re^s$$

Evaluate $\frac{\partial z}{\partial s} = f'(w) \cdot w'$ for $r = 1$ and $s = 0$.

$$\begin{aligned}\left.\frac{\partial z}{\partial s}\right|_{r=1,\, s=0} &= -5\left(8s + 5re^s\right) \\ &= -5\left[8(0) + 5(1)e^{(0)}\right] \\ &= -25\end{aligned}$$

Therefore, $\left.\frac{\partial z}{\partial r}\right|_{r=1,\, s=0} = 395$ and $\left.\frac{\partial z}{\partial s}\right|_{r=1,\, s=0} = -25$.

$z = f(w)$
$w = g(x,y)$

$x = 3r^3 - s^2$
$y = re^s$

$g_x(3,1) = -4$
$g_y(3,1) = 2$

$f'(7) = -4$
$g(3,1) = 7$

$z = f(w) \qquad z' = f'(w) \cdot w'$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(4y - 5x)$$
$$= -5$$

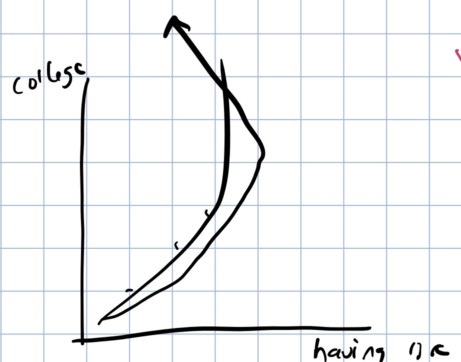
$$\nabla f\big|_{(1,4)} = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j}$$
$$= -5\hat{i} + 4\hat{j}$$

$(5x)\hat{i} +$

$$\frac{\partial f}{\partial x} = xy^2 \qquad \frac{\partial f}{\partial y} = xy^2$$
$$= y^2 \qquad = 2xy$$

$$\Delta f\big|_{(7,5)} = y^2 i + 2xy\, j$$

$$= 5^2 i + 2(7)(5) j$$
$$25i + 2(35) j$$
$$25i + 70j$$



you don't know it
for real

$2x$

$\frac{\partial f}{\partial x}\left(x^3+y^3-4z^2+z\ln x\right)$ $\qquad$ $\frac{\partial f}{\partial y}(\cdots)$ $\qquad$ $\frac{\partial f}{\partial z}(\cdots)$

$= 3x^2+0+0+z\ln x$ $\qquad$ $= 0+3y^2$ $\qquad$ $= 0+0+8z+x$

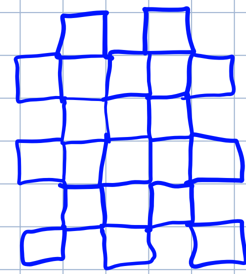
$= 3x^2+\frac{z}{x}$ $\qquad$ $= 3y^2$ $\qquad$ $= -8z+ \quad \ln x$

$2z$

z

$2 \ln x$

$\frac{1}{x}$



$\Delta f\Big|_{(1,4,5)} = 3x^2 i + 3y^2 j + (z\ln + 8z + x)k$

$= 3(1)^2 i + 3(4)^2 j + (5\ln + 8(5)+1)k$

take partials, then plug point. $\Delta f\Big|_{x,y}$

you have vector A. find $|A|$ $\qquad i+j+k$

then use formula $u=\frac{A}{|A|}$, u_x "i" and u_y "j"

$D_Af = \frac{\partial f}{\partial x}u_x + \frac{\partial f}{\partial y}u_y$

$\frac{d}{dx}\left[\frac{f(x)}{f(y)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

$g(x,y) = \frac{x-y}{xy+3}$ $\qquad P_0\ (2,-2)$ $\qquad u = 3i+4j$

$\frac{\partial f}{\partial x}\left(\frac{x-y}{xy+3}\right) = \frac{\cancel{xy+3}(1)-(x-y)(y)}{(\cancel{xy+3})^2} = \frac{xy+3-xy-y^2}{(xy+3)}i$